Research Paper

Decoding Market Efficiency: A Multifractal Approach to Financial Market Dynamics

Roja Putera Mahyudin[1]*, Lamsah Lamsah[2]
[1] Department of Management, Islam Kalimantan University

Abstract

Financial markets were traditionally analyzed using the Efficient Market Hypothesis (EMH), which argued that asset prices fully reflected all available information. This led to the belief that price movements were random, making it impossible to consistently outperform the market. However, emerging research had highlighted that markets could exhibit multifractality, where inefficiencies existed across different time scales, challenging the traditional EMH framework. Our study built on these insights by investigating the market efficiency of several asset classes, including stock market indices, cryptocurrencies (Bitcoin, Ethereum, USDT, and Binance), commodities (gold and crude oil), fixed income (U.S. 10-Year & 30-Year Treasury Bonds), and foreign exchange (EUR/USD & GBP/USD). Utilizing the Multifractal Detrended Fluctuation Analysis (MFDFA) method, we analyzed the multifractal properties and efficiency of each asset class. The results revealed that traditional assets exhibited monofractality, adhering more closely to the EMH. In contrast, cryptocurrencies displayed significant inefficiencies, likely due to their volatility, market structure, and susceptibility to external shocks. The findings provided valuable insights into portfolio diversification and risk management, suggesting that inefficiencies in certain asset classes might present both opportunities and risks for investors.

Keywords: *Efficient Market Hypothesis; Multifractal; Cryptocurrency*

INTRODUCTION

The increasing complexity and diversity of global financial markets have raised important questions about the efficiency of various asset classes. While traditional assets like stocks and bonds have been extensively studied under the Efficient Market Hypothesis (EMH) framework, comparison between traditional and emerging assets offer new challenges for understanding market behaviour (Nazlioglu et al., 2023). According to the EMH, in an efficient market, asset prices fully reflect all available information, making it impossible to consistently outperform the market using historical price data (Fama, 1970). However, the degree of efficiency can vary significantly across different markets and asset types.

Recent advances in econometric techniques, such as Multifractal Detrended Fluctuation Analysis (MFDFA), allow researchers to explore complex financial time series that exhibit multifractality—a property indicating long-term correlations, non-linear dependencies, and volatility clustering (Gorjão et al, 2022). MFDFA has proven useful in detecting inefficiencies in asset classes with irregular price movements, making it an ideal tool for analyzing a wide range of assets.

The objective of this study is to examine whether these asset classes conform to the assumptions of the EMH or whether they display inefficiencies that investors could potentially exploit. By applying the MFDFA method, we aim to detect multifractality in the price series of these assets, which would indicate the presence of long-term dependencies and market inefficiencies. In doing so, we contribute to the literature by providing a comprehensive cross-asset comparison of efficiency, with particular focus on more established assets and emerging assets volatility.

Kilic et al. (2023) found that Bitcoin and Ethereum are not efficient in long-term period, while Mnif et al. (2022) found that Bitcoin is the most efficient between conventional cryptocurrency in short-term horizon. Kakinaka and Umeno (2022) the markets exhibit different multifractality before and after economic disruption. There also evidence that conventional method can give misleading result because of volatility clustering. Mahyudin et al. (2023) provide insights into the latest MFDFA method for testing market efficiency, indicating long-term correlation. The efficacy of this method is demonstrated through the analysis of 20 years of data from a large sample of companies in the US and China.

The primary research question of this study is: Do traditional and emerging asset classes, particularly cryptocurrencies, conform to the assumptions of the Efficient Market Hypothesis, or do they exhibit inefficiencies that could be exploited by investors? To address this, we applied the MFDFA method to detect multifractality in the price series of various asset classes, which indicated the presence of long-term dependencies and market inefficiencies. The research objectives were threefold: (1) to compare the efficiency of traditional versus emerging assets; (2) to identify inefficiencies in emerging assets such as cryptocurrencies; and (3) to provide a comprehensive cross-asset comparison, focusing on how volatility impacted market efficiency across different time horizons. In doing so, we contributed to the literature by offering insights into market.

LITERATURE REVIEW

The concept of a random walk, which is a central component of the efficient market hypothesis (EMH), assumes that prices in financial markets move in a random and unpredictable manner (Malkiel & Fama, 1970). The notion of a random walk is often used to support the idea of market efficiency, as it suggests that prices fully reflect all available information, making it impossible to profit from market inefficiencies (Fama, 1970). Therefore, any investment strategies using financial statements, news reports, historical price, or insider information would not help in finding undervalued assets.

However, the efficient market assumptions may not hold due to various aspects such as unfavorable environment (Karampinis & Hevas, 2011), market volatility (Shiller, 2015), and investor rationality (Tversky & Kahneman, 1992). These factors affect financial market behavior to deviate from efficient market assumptions. The efficient market should be stable, price is fair, and intrinsic value aligns with the price. Although the price can be undervalued or overvalued as long as the errors/deviations are random. Investor rationality play an important role in this theory, which many empirical studies have problem with.

The theory can be seen from different point of view. Fractal market hypothesis brings idea of market efficiency can be tested through randomness of price. Rational investors get all information available to invest 'rationally' which create random walk in price movements. This randomness concept is similar to fractals. Fractal is a geometric shape that have self-similarity in various scales. This shape can be either zoomed in or out to get an exact same outcome. Fractal can contain a randomness or chaotic behavior inside the system. This system called deterministic chaos, which is used to describe a chaotic pattern or randomness governed by deterministic laws.

One of many interesting fractals out there that can explain how financial market works is bifurcation diagram. Bifurcation diagram represent stable values and unstable values (chaotic/randomness) from a mathematic function. Logistic map with this function $X_{n+1} = r X_n + (1 - X_n)$ generate 3 phases which are, fixed points (stable), oscillations periods, and chaotic attractors (unstable). This is a chaotic pattern that occurs from a deterministic law or mathematical function. Logistic map apparently has connection to the Mandelbrot set a famous fractal that found by Benoit Mandelbrot in 70s. Bifurcation diagram is a part of the Mandelbrot set with the same structure of 3 phases in 2 dimensions.

Fractal Market Hypothesis explain financial market from fractal perspective. There are several phases in the financial market including efficient/random walk that indicated by chaotic attractors. In bifurcation diagram, the chaotic attractors occur in the third phase when *r* value 3.57. In the context of market efficiency, financial market efficiency can be calculated using fractal detection in price fluctuation. Usually, in the traditional method, if the market exhibit random behavior then it is efficient. However, using fractal method, if the market exhibit monofractal then it is efficient and if the market exhibit multifractal then it is not efficient. For a more detailed understanding of this framework, please refer to the theoretical framework section below.
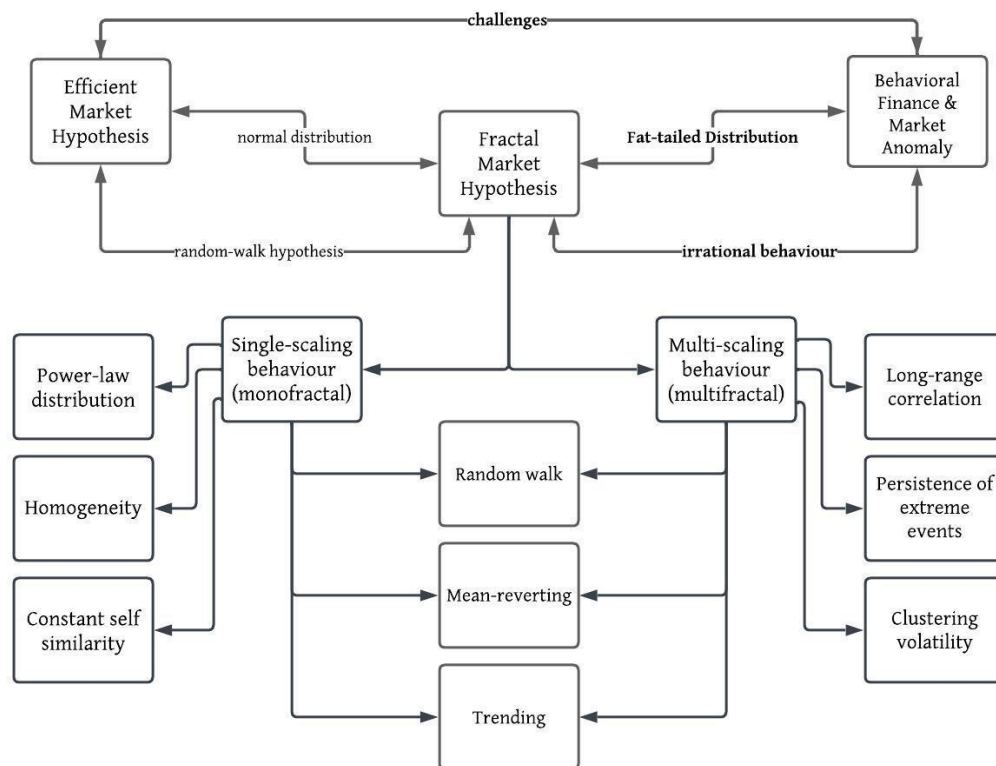

Figure 1. Theoretical Framework

In the realm of financial markets, the distinction between monofractal and multifractal dimensions is important when considering market efficiency. A market that can be described by a single fractal dimension is referred to as a monofractal system and vice versa. In essence, this implies that the market exhibits a uniform and consistent degree of self-similarity or scaling across various time frames. The monofractal market is that its price movements follow a single, consistent pattern of behavior over time, as indicated by a single Hurst exponent in the context of fractal analysis (Shrestha, 2019).

Conversely, when a market cannot dictated by a single fractal dimension, it is called a multifractal system (Gorjão et al., 2022). In multifractal markets, the degree of self-similarity and scaling varies across different time scales or contexts. This implies that market dynamics are characterized by a multitude of underlying processes, each with its distinct scaling properties. In practical terms, multifractal behavior suggests that market efficiency may fluctuate significantly across different time frames or in response to changing conditions.

There are several studies with the latest method to test market efficiency, for example a study by Kilic et al. (2023) that provide evidence using Fourier non-linear quantile unit root approach (FNQKS), Diniz-maganini et al. (2023) that compare foreign exchange rates efficiency in BRICS

countries, Mensi et al. (2022) that test asymmetric multifractality on several assets classes including cryptocurrency and commodities. However, the effectiveness of these methods still debated. Gorjão et al. (2022) developed a multifractal analysis using Empirical Mode Decomposition (EMD) the Hilbert-Huang decomposition time series for detrending tools in python. The reliability and effectiveness of this method were tested by Mahyudin et al. (2022) through a large sample of data spanning more than 23 years, covering 343 US companies and 148 Chinese companies. In conclusion, the method effectively demonstrates its robustness in evaluating market efficiency through extensive, long-term data from both US and Chinese companies.

RESEARCH METHOD

This research using Multifractal Detrended Fluctuation Analysis (MFDFA) to identify market efficiency, then integrating multifractal analysis into the value relevance model. The latest advancement of MFDFA by Gorjão et al (2022) is used in this research. It utilizes Empirical Mode Decomposition (EMD) for detrending technique and efficient python code for calculations. Daily close price data was retrieved from Yahoo Finance from 2000-12-31 until 2024-8-31 resulting approximately 5953 sample size data for each assets. This study focuses on five major asset classes:

1. Stock Market Indices (S&P500, Nasdaq, Russel 2000, Hangseng): These represent a diversified portfolio of publicly traded companies, typically considered efficient due to high liquidity and wide participation.
2. Cryptocurrencies (BTC, ETH, USDT, and BNB): Known for extreme price volatility and speculative trading, cryptocurrencies offer a novel avenue for testing market efficiency.
3. Commodities (Gold and Crude Oil): Traditionally seen as safe havens or economic indicators, commodities like gold and oil are subject to macroeconomic factors and geopolitical risks, which can affect their price behavior.
4. Fixed Income (U.S. 10-Year and 30-Year Treasury Bonds): As one of the most stable asset classes, government bonds provide a benchmark for risk-free returns and are usually considered highly efficient.
5. Foreign Exchange (EUR/USD and GBP/USD): The forex market is the largest and most liquid in the world, but currency pairs may still exhibit inefficiencies due to central bank interventions, macroeconomic data releases, and speculative behavior.

MFDFA studies the variance of the fluctuations of a given process by considering increasing in segments of a time series (Gorjão et al., 2022) [p. 2]. First, Detrend the time series data to remove any non-stationary trends or drifts. A common method is polynomial detrending, where you fit a polynomial to the data and subtract it to get the residuals.

$$Y = \sum_{k=1}^{i} (Xk - \mu X), \text{for } i = 1, 2, \ldots, N, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Divide the detrended time series into smaller, non-overlapping segments. The length of each segment depends on the size of your dataset and the level of fluctuations you want to analyze.

$$F(v,s) = 1/2 \sum_{k=1}^{i} [(Y(v-1)s + i) - (Y(v-1)s + i)]^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For each segment, calculate the root-mean-square fluctuations *Fq(s)* for different time scales (N). The time scales can be chosen in a logarithmic or linear manner.

$$Fq(s) = \{1/Ns \sum_{v=1}^{Ns} [F(v,s)]^{q/2}\}^{1/q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Compute the generalised Hurst exponent *h(q)* for each segment. The hurst exponent is determined using "the slope of *Fq(s)* curve in the log-log plots" [p. 2]. If the data have monofractal properties, then the *h(q)* is independent of q (not using various q). For monofractal *q=2,* multifractal *q = {-10;10}*

$$Fq(s) \sim s^{\,h(q)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$
$$\tau(q) = qh(q) - 1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

Calculate multifractal scaling exponent $\tau(q)$ by averaging the local scaling exponents from all segments. This is the basic of MFDFA by Gorjão et al. (2022), which use Empirical Mode Decomposition (EMD) and various multifractal *q = {-10;10}* instead of *q=2.* Last, the generalized Hurst exponent is determined using "the slope of *Fq(s)* curve in the log-log plots"

Monofractal   Multifractal
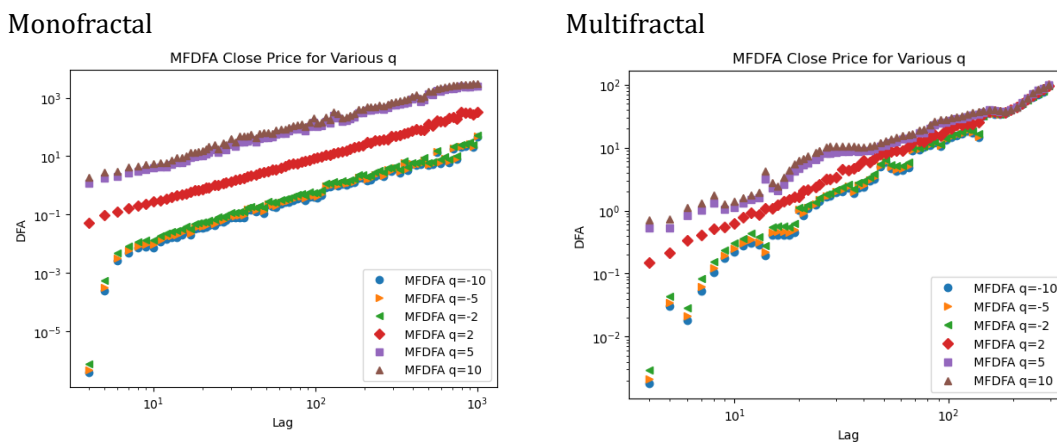
Figure 2. Example of MFDFA Result

The Empirical Mode Decomposition (EMD) the Hilbert-Huang decomposition time series will replace the traditional detrended method. For example, polynomial fittings is replaced by Intrinsic Mode Function (IMFs). This process will contain the residual trend of the data and transform the non-stationary series into stationary one. From the graph above, the slope will dictate whether monofractal or multifractal properties taken place. This analysis method provides better result for market efficiency analysis than previous MFDFA.
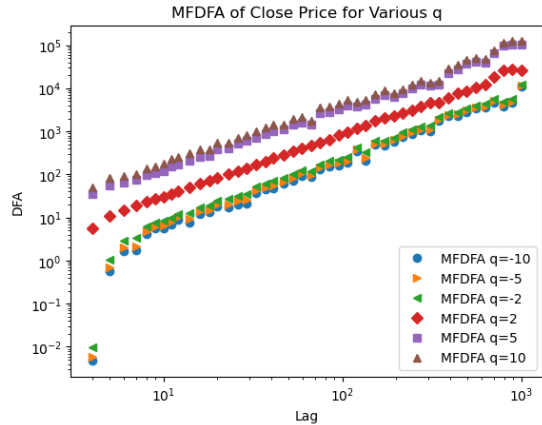
FINDINGS AND DISCUSSION

This research employs the Multifractal Detrended Fluctuation Analysis (MFDFA) to investigate the efficiency of several financial markets. The results provide valuable insights into the behavior of different asset classes, from traditional stock indices to cryptocurrencies and commodities.

The analysis reveals that major stock indices such as the S&P 500, Nasdaq, Russell 2000, and the Hang Seng index exhibit characteristics of efficient markets. According to the Efficient Market Hypothesis (EMH), efficient markets reflect all available information, and prices adjust rapidly to new data. The findings from MFDFA show a minimal level of long-range correlations in these indices, which is indicative of high market efficiency. The fractal patterns observed in these indices suggest that they operate within a framework where price movements are unpredictable and respond to new information almost immediately, confirming the assumptions of market efficiency.
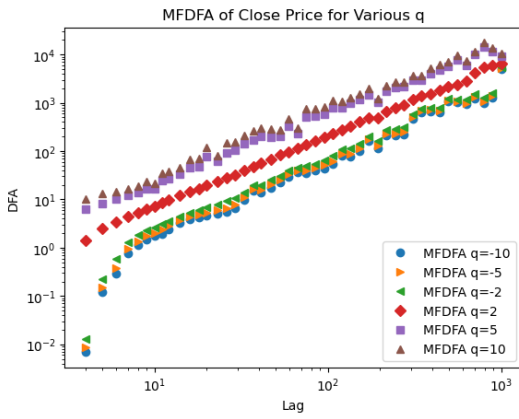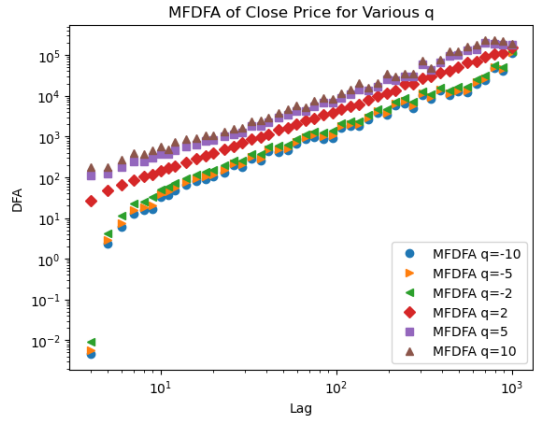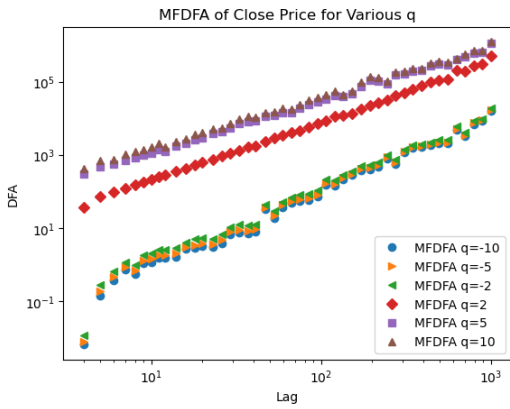
^GSPC (S&P500)

MFDFA of Close Price for Various q

^IXIC (Nasdaq)

MFDFA of Close Price for Various q

^RUT (Russel 2000)

MFDFA of Close Price for Various q

^HSI   (Hang   Seng)

MFDFA of Close Price for Various q

BTC-USD
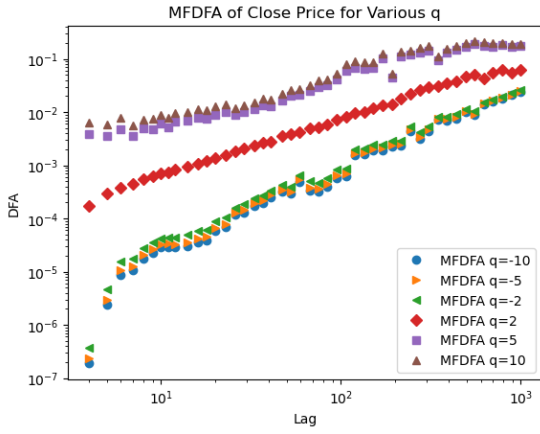
MFDFA of Close Price for Various q
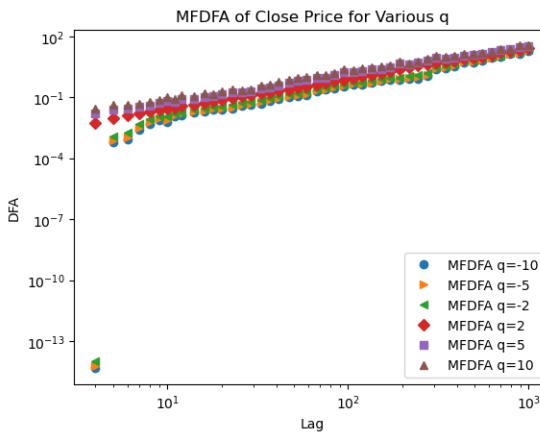
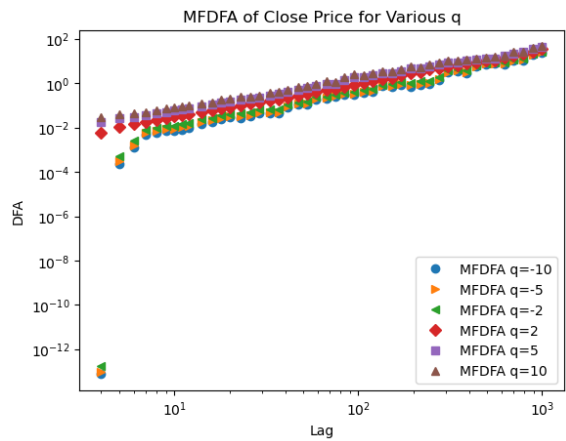ETH-USD

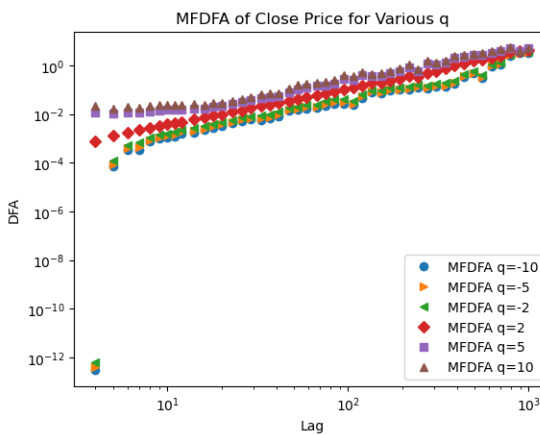MFDFA of Close Price for Various q
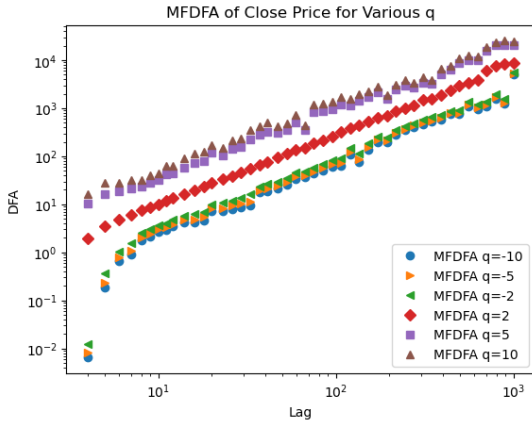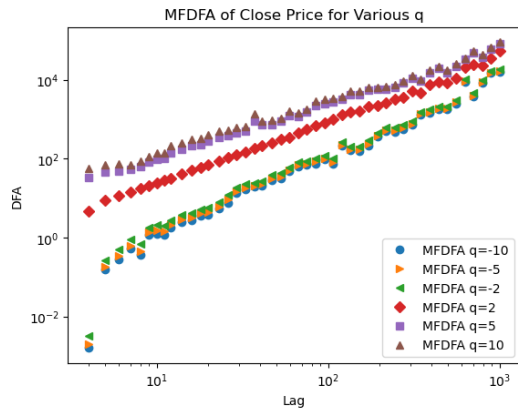
USDT-USD
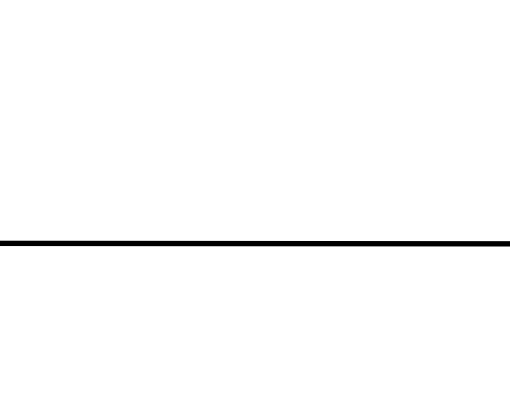
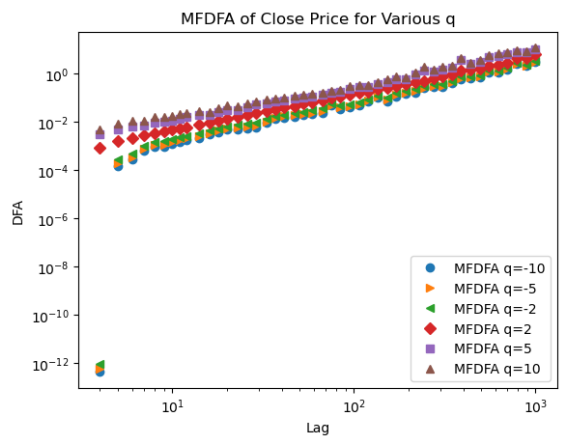BNB=USD
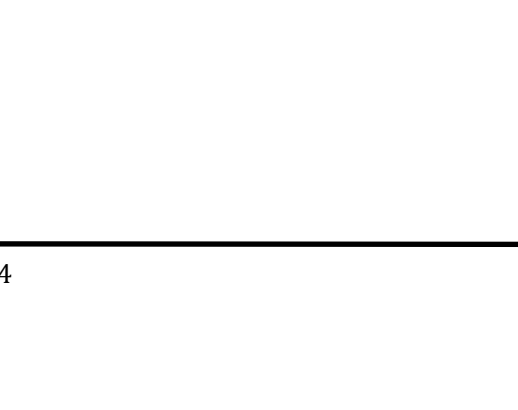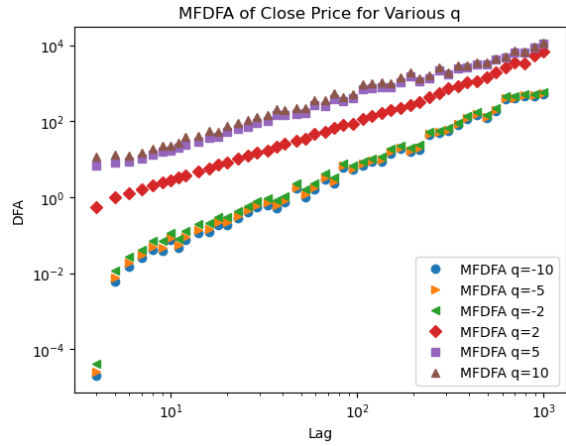
^TYX (US Treasury Bond 30 yrs)

^TNX (US Treasury Bonds 10 Yrs)
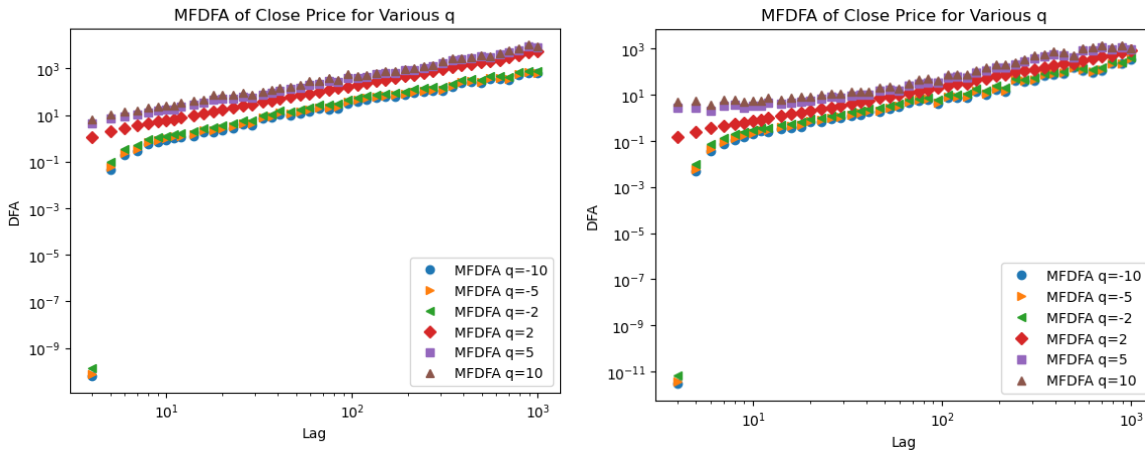

EURUSD=X

GBPUSD=X

GC=F (Gold)

CL=F (Crude Oil)

On the contrary, the analysis of cryptocurrencies like Bitcoin, Ethereum, USDT, and Binance Coin suggests significant inefficiency in these markets. The presence of long-term correlations and persistent fractal structures points to inefficiencies that may be due to a combination of factors such as immature market structure, high volatility, and speculative trading. The inefficient nature of these assets highlights their deviation from traditional financial markets, suggesting that cryptocurrency prices are driven by factors like market sentiment, speculative bubbles, and lack of regulatory oversight, rather than the fundamental information that drives efficient markets.

The results indicate that both the 30-year and 10-year U.S. Treasury bonds display high efficiency. This is consistent with the idea that government bonds are among the most liquid and widely traded assets in the financial markets, with prices that quickly incorporate available information. The minimal fractal behavior in these bonds points to stable and efficient pricing mechanisms, driven largely by interest rate expectations and macroeconomic factors, aligning them with the characteristics of efficient markets.The Forex markets, particularly the EUR/USD and GBP/USD pairs, also exhibit signs of efficiency. Given the vast size and liquidity of the Forex market, the lack of significant long-range correlations suggests that exchange rates adjust quickly to new information, reflecting the efficient flow of data in global currency markets. These results reinforce the notion that Forex markets, with their high trading volumes and round-the-clock activity, are highly efficient. The commodities market, represented by gold and crude oil, is shown to be efficient as well. These assets, while subject to external shocks (such as geopolitical events or supply-demand imbalances), generally operate within efficient market dynamics. The absence of strong long-range correlations in their pricing patterns suggests that commodity markets rapidly incorporate new information, which aligns with the behavior expected in efficient markets.

The findings from this research highlight a clear divide in efficiency between traditional financial markets (stocks, bonds, forex, commodities) and emerging or alternative assets like cryptocurrencies. The efficiency in traditional markets can be attributed to their long-standing infrastructure, regulation, and the vast amount of public information available. In contrast, the inefficiency in cryptocurrency markets can be explained by their volatility stage, lack of institutional oversight, and speculative nature. These results align with previous studies that also identified inefficiencies in cryptocurrency markets. For instance, Shrestha (2021) found that Bitcoin returns are multifractal, indicating a lack of efficiency in its market. Similarly, Mnif et al. (2020) noted that most cryptocurrencies were multifractal before the COVID-19 pandemic but became more efficient afterward, suggesting that market conditions can influence efficiency. Conversely, our research emphasizes the persistent inefficiencies in cryptocurrencies, which contrasts with the improvements noted by Mnif et al. (2020) after the pandemic.

In traditional markets, our findings resonate with the work of Miloş et al. (2020), who reported long-range correlations in stock indices, supporting the notion that these markets may not be entirely efficient and are still maturing. Furthermore, Mensi et al. (2022) highlighted that inefficiencies were heightened during the COVID-19 pandemic, except for Bitcoin and gold, which adds complexity to our understanding of market dynamics. Overall, our research underscores the importance of asset selection in portfolio management. Investors seeking exposure to more predictable price movements may gravitate toward efficient markets, whereas those looking for opportunities in speculative and inefficient markets may turn to cryptocurrencies, albeit with greater risk.

In conclusion, the latest MFDFA approach provides robust evidence supporting the efficient nature of traditional asset classes while highlighting the inefficiencies present in the cryptocurrency market. This has significant implications for both investors and policymakers, as it stresses the need for caution when dealing with newer, less efficient asset classes. The novelty of our study lies in its comprehensive comparison of various asset classes, revealing persistent inefficiencies in cryptocurrencies and contributing to the broader discourse on market efficiency in the context of emerging assets.

CONCLUSIONS

This research utilizing the latest version of the Multifractal Detrended Fluctuation Analysis (MFDFA) provided a robust framework to uncover the fractal properties of these markets, offering deeper insights into their efficiency dynamics compared to earlier methods. The advanced capabilities of this method allowed for more accurate detection of multifractal behaviour. The result demonstrated that traditional asset classes such as stock indices (S&P 500, Nasdaq, Russell 2000, Hang Seng), U.S. Treasury bonds (10-year and 30-year), forex pairs (EUR/USD and GBP/USD), and commodities (gold and crude oil) exhibit characteristics of efficient markets. In contrast, cryptocurrencies like Bitcoin, Ethereum, USDT, and Binance Coin were found to be inefficient, highlighting a key distinction between emerging digital assets and traditional markets. The MFDFA method provided a robust framework to uncover the fractal properties of these markets, offering insights into their efficiency dynamics. These findings have important implications for investors, indicating that traditional assets may provide more stable and efficient investment opportunities, while cryptocurrencies are subject to inefficiencies and higher volatility.

LIMITATION & FURTHER RESEARCH

One important area for further research is the exploration of how market efficiency evolves over time. Financial markets are not static; they are influenced by external shocks, macroeconomic events, technological advancements, and policy changes. Conducting a longitudinal study using MFDFA across different time periods, such as during financial crises, market booms, or global events, would provide valuable insights into how efficiency changes under different conditions. This approach would also allow for a better understanding of whether certain asset classes, such as cryptocurrencies or commodities, become more or less efficient over time. Temporal analysis could help identify whether specific markets are prone to long-term inefficiency and which factors contribute to a shift toward or away from efficient behavior.

While this study employed MFDFA, future research could benefit from applying alternative multifractal methods, such as wavelet-based multifractal analysis or multifractal cross-correlation analysis (MFCCA). These approaches may capture different aspects of fractal behavior in financial markets, offering a more nuanced understanding of market dynamics. For instance, wavelet-based methods could provide better time-frequency localization, allowing researchers to investigate how short-term and long-term market efficiency evolve across various asset classes. Additionally,

MFCCA could reveal the interdependencies between asset classes and whether the inefficiencies in one market (e.g., cryptocurrencies) spill over into other markets (e.g., traditional stock indices). By using complementary methods, future studies could enrich the findings and help cross-validate results obtained from MFDFA, providing a more comprehensive framework for understanding market efficiency across different financial landscapes.

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