A Binary Integer Linear Programming Approach for Risk Minimization of a Multi-Mode Resource-Constrained Project Scheduling Problem with Discrete Time-Cost-Quality-Risk Trade-Off

Ricardo C. Alindayu II, Ronaldo V. Polancos, Rosemary R. Seva

Abstract

The multi-mode resource-constrained project scheduling problem (MRCPSP) allows project managers to assign schedules and limited resources to project activities while minimizing total duration. This time objective has been extended to include the trade-offs of cost and quality called the discrete time-cost-quality trade-off problem (DTCQTP) to provide project managers an overview of indicators affecting project performance. However, the impacts of the COVID-19 pandemic have led project managers and consultants to assess their projects’ risks regarding scheduling and resource assignment decisions. Hence, this paper aims to extend the MRCPSP and the DTCQTP to include risk at the resource level to highlight the importance of hiring the proper resources in project scheduling. A binary integer linear programming model named the multi-mode resource-constrained project scheduling with discrete time-cost-quality-risk trade-off (MRCPSP-DTCQRT) was developed with risk minimization as the objective function. A case study from prior literature was used to illustrate the model using the open-sourced Python-MIP package, which uses the branch-and-cut methodology for generating optimal solutions. A set of schedules that either prioritize time, cost, quality, risk, or a balance among the four was generated for the use of the project manager to make decisions based on the current situation. Future research may extend the model further to include resource skills, test large-scale case studies, and use other methodologies such as metaheuristics and machine learning to arrive at optimal solutions within reasonable computing time.

Keywords: Project Scheduling, Project Management, Integer Linear Programming, Multi-Mode Resource-Constrained Project Scheduling Problem, Discrete Time-Cost-Quality-Risk Trade-Off

INTRODUCTION

Project managers and consultants aim to create value for their clients by planning and executing time-bound projects that achieve predetermined objectives. These projects have a set of activities and resources that the project manager must allocate properly to meet the deadline based on precedence and resource constraints. The multi-mode resource-constrained project scheduling problem (MRCPSP) was created to generate an optimal schedule of activities that minimizes project duration while satisfying logical constraints (Afshar-Nadji, 2014). In this model, activities can be executed in more than one way or "mode" which the project manager selects to complete the task (Hartmann & Briskorn, 2021). While the MRCPSP is sufficient for practical solutions, it does not consider other objectives that a project manager might need to make better decisions. Research has expanded the MRCPSP from the single objective of makespan minimization to include other essential objectives that also affect the project's performance.

The discrete time-cost trade-off problem (DTCTP) is the most common extension to the MRCPSP that recognizes more than one objective to the MRCPSP. The DTCTP recognizes the trade-off relationship that exists between project time and cost. In general, reducing a project’s duration increases costs and vice versa. As a result, a set of non-dominated solutions to the problem may be generated and used by the project manager to make a sound decision based on their preferences or current situation. However, time and cost alone do not provide the full picture, as they also impact the project’s overall quality. As a result, the DTCTP now includes project quality, which refers to the output's conformity to specified standards (Pour et al., 2012). The discrete time-cost-quality trade-off problem (DTCQTP) refers to these three parameters as the “iron triangle” (Pollack et al., 2018). Constant project cost and time results in lower quality projects, while increasing costs and time results in higher quality projects. When creating optimal schedules, there is a clear three-way trade-off between time, cost, and quality (Liberatore & Pollack-Johnson, 2013).
The COVID-19 pandemic has disrupted enough projects for managers to re-ignite the concept of risk in project scheduling (Fauziyah & Susanti, 2022). The "devil's quadrangle" incorporates time, cost, quality, and risk into the iron triangle concept. In this context, risk refers to the likelihood of an activity being repeated due to a lack of output. As a result, risk must be included in the RCPSP's trade-off objectives. This will allow project managers to better understand their planned schedules by calculating project risk alongside time, cost, and quality. This paper has three objectives: (1) a literature review identifies the time-cost-quality-risk trade-off relationship, (2) a binary integer linear programming model demonstrates the MRCPSP-DTCQRTP, and (3) a case study from prior literature demonstrates the MRCPSP-DTCQRTP.

LITERATURE REVIEW

A. Multi-Mode Resource-Constrained Project Scheduling Problem

The resource-constrained project scheduling problem (RCPSP) generates an optimal schedule that performs a set of non-preemptive activities that satisfies the given precedence relations and resource constraints and minimizes the total project time or "makespan" (Hartmann and Briskorn, 2021). Each activity in the RCPSP requires a set of resources for its processing. They are generally of two types: renewable and non-renewable. Renewable resources such as manpower may be split during a project which allows resource sharing and hence, can be incorporated into the scheduling process. In contrast, money is usually a non-renewable resource and cannot be replenished once consumed (Chakrabortty, Sarker, and Essam, 2020).

In most cases, the objective when solving the RCPSP is to find the shortest possible duration of the project, called the "makespan," while the availability of renewable resources is considered given (Young, Feydy, and Schutt, 2017). However, the importance of resources is often overlooked when modeling using the RCPSP. In project scheduling, hiring and managing the right resources are vital to the project's success. Momoh et al. (2010) provide a literature review on critical factors that cause ERP system implementations projects to fail from more than a decade's worth of literature. Alternatively, Nguyen & Hadikusumo (2017) emphasize how human resources positively impact project success when consulting with 800 personnel. Hence, human resources play a crucial role in the RCPSP, which is not reflected in the classical project scheduling problem.

Given the very simplistic nature of the RCPSP, there have been efforts to amend portions of the standard model to adapt to realistic situations encountered in the industry by project managers. Hartmann and Briskorn (2021) provide a literature review of the variants and extensions of the RCPSP that aim to respond to the growing interest in the problem. While their paper highlights multiple variants from over a decade of literature, only the multi-mode RCPSP (MRCPSP) and discrete time-cost tradeoff problem (DTCCTP) shall be highlighted in this literature review since they are widespread generalizations of the variants that exist.

A generalization of the RCPSP called the multi-mode RCPSP (MRCPSP) was developed to account for the many possible resources that a project manager can assign to activities depending on their requirements. Researchers recognize that several execution "modes" exist in the literature to accomplish an activity where each available resource represents a possible execution mode for an activity (Afshar-Nadjafi, 2014; Hartmann and Briskorn, 2021). This provides flexibility on the part of the project manager to select the best execution mode that may not have been considered in the RCPSP due to its simplistic nature (Afshar-Nadjafi, 2014).

Common to the research area of the MRCPSP is the recognition that multiple execution modes in the form of resources can be allocated to a project to fulfill specific objectives. For example, a particular resource may accomplish an activity at a shorter duration than another resource. Hence, these resources represent two execution modes that a project manager may select depending on their preference or goal. This is a generalization of the original RCPSP, where resources are assumed to be equivalent, which is not the case in real life. Hence, researchers have commonly used the MRCPSP as a basis for their schedule modeling.
B. Discrete Time-Cost Trade-off Problem and its Extensions

The use of multiple resources for a project does affect not only the total project time but also the project cost. In general, shorter durations would entail higher costs, presenting a trade-off problem where it is highly preferred that both are minimized simultaneously. This relationship is visualized in Figure 1. This extension of the MRCPSP to include cost is called the discrete time-cost trade-off problem (DTCTP) forming a bi-objective optimization problem (El-Abbasy, Elazouni, and Zayed, 2017).

The Discrete Time-Cost-Quality Tradeoff Problem (DTCQTP) is perhaps one of the more common extensions of the traditional DTCTP in project scheduling research. Khalili-Damghani et al. (2015) defines quality as the percent conformance to a specified parameter set by the project manager. Tran et al. (2015) similarly adapt the quality parameter, highlighting its strong correlation to time and cost. For their study, higher costs would lead to shorter project durations and decreased output quality. Previous studies before Khalili-Damghani have used quality in their optimization models using empirical data. Liberatore & Pollack-Johnson (2013) incorporate quality as part of the time-cost trade-off in project scheduling, emphasizing a continuous relationship between the three parameters. This curve may be projected onto a 2D graph, as shown in Figure 2. Kannimuthu et al. (2019) also focus on the time-cost-quality trade-off in the MRCPSP using data from real-life construction projects in India.

Nguyen et al. (2021) designed a novel metaheuristic algorithm for the DTCQTP called the multiple objective whale optimization (MOWO). Their study applied the MOWO to solve the optimum TCQ parameters for a non-repetitive construction project. Shortening the time to perform a project (called “crashing”) increases incurred costs and decreases the overall quality of the project. This relationship coincides with Khalili-Damghani et al. (2015) and Tran et al. (2015).

Tran et al. (2018) extend the DTCTP to include risk into the trade-off model, now referred to as the DTCRTP. In their paper, the risk is defined as the probability that a schedule delay may arise from reducing
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the total makespan of the project due to loss of total float and lack of resources. Another definition of risk exists in the literature focusing on the risk attitudes of project managers when designing a project schedule (Subulan, 2020). Both Tran et al. (2018) and Subulan (2020) have varying risk definitions. However, Maghsoudlou et al. (2017) refer to risk as the probability of a resource reworking an activity if it does not conform to the required standards which shall be adopted for this paper.

![Figure 3. Proposed time-cost-quality-risk curve](image)

There have been efforts to combine the DTCQTP and DTCRTP into a single problem. Mahmoudi & Feylizadeh (2018) extend the DTCTP to include quality and risk in the trade-off, thus referred to as the DTCQRTP. In general, longer times would result in higher quality, lower risk, and lower cost, highlighting the trade-off relationship between the four parameters. This may be represented as a 2D graph in Figure 3, where risk curves are added to the visualization presented in Figure 2. The problem includes the usual decision variables for the DTCTP, including quality and risk. However, they are represented as additional costs in the model apart from the standard percentage representation in the literature. On the other hand, Amoozad Mahdiraji et al. (2021) created a novel DTCQRTP using stochastic information derived from project managers to achieve the intended practicality of scheduling problems. However, the values of time, cost, quality, and risk are assigned to activities without regard to the importance of resources, as established in the previous research, providing an opportunity to expand their work to incorporate resource-focused decision variables.

**RESEARCH METHODOLOGY**

**C. Model Formulation**

The MRCPSP-DTCQRT formulation is based on research conducted by Pollack-Johnson & Liberatore (2006), Maghsoudlou et al. (2016) and Amoozad Mahdiraji et al. (2021). This research combines their contributions into a singular model with the addition of a non-preemptive constraint that limits resources to execute only one activity at a time and the inclusion of risk as a trade-off objective. Given this, the MRCPSP-DTCQRT has the following assumptions:

1. The activities (n) of the project are numbered from 0 to n + 1, with 0 as a dummy start activity and n + 1 as a dummy end activity.
2. All resources to be used in the project are manpower and are available for use throughout the entire project depending on availability constraints.
3. Time, cost, quality, and risk (TCQR) values are dependent on the resource’s ability to perform an activity.
4. Each activity may be executed in several modes, with each mode representing a resource’s ability to complete the activity at the set time, cost, quality and risk.
5. Pre-emption of activities is not allowed while execution is ongoing.
6. The assigned resource cannot be replaced during execution of an assigned activity.
7. A resource cannot execute two or more activities at the same time.
8. The time between activities is negligible.
Like other models in the RCPSP literature, the following parameters and decision variables are defined for the MRCPSP-DTCQRT model:

- \( n \) Number of activities for the project
- \( A \) Set of precedence constraints
- \( I \) Set of activities
- \( J \) Set of resources
- \( T \) Set of time periods
- \( d_{ij} \) Duration of resource \( j \) for activity \( i \)
- \( c_{ij} \) Cost of resource \( j \) for activity \( i \)
- \( q_{ij} \) Quality of resource \( j \) for activity \( i \)
- \( r_{ij} \) Risk of resource \( j \) for activity \( i \)
- \( T_{ub} \) Upper bound of the total project time
- \( C_{ub} \) Upper bound of the total project cost
- \( Q_{lb} \) Lower bound of the weighted average project quality
- \( t_i \) Scheduled start time for activity \( i \)
- \( x_{ijt} \) 1 if activity \( i \) is executed with resource \( j \) at start time \( t \), 0 if otherwise

It should be noted that for this model, the indices of activities, resources and time periods are combined into a single binary variable for simplicity in expressing constraints and objective functions. This research paper simplifies the previous authors’ variables by consolidating everything into a single expression, \( x_{ijt} \).

And finally, the MRCPSP-DTCQRT integer linear programming model is presented with the following set of equations:

\[
\text{Minimize } R = \frac{r_{ij} \cdot x_{ijt}}{n} \tag{1}
\]

\[
t_{n+1} \leq T_{ub} \tag{2}
\]

\[
\sum_{t=0}^{T} \sum_{j=1}^{J} \sum_{i=1}^{I} c_{ij} \cdot x_{ijt} \leq C_{ub} \tag{3}
\]

\[
q_{ij} \cdot x_{ijt} \geq Q_{lb} \tag{4}
\]

\[
\sum_{j=1}^{J} \sum_{t=0}^{T} t \cdot x_{ijt} - \sum_{j=1}^{J} \sum_{t=0}^{T} t \cdot x_{ijt} \geq \sum_{j=1}^{J} \sum_{t=0}^{T} x_{ijt} \cdot d_{ij}, \forall (i,k) \in A, \forall j \tag{5}
\]

\[
\sum_{j=1}^{J} \sum_{t=0}^{T} x_{ijt}, \forall i \in I \tag{6}
\]

\[
\sum_{t=0}^{T} x_{ijt} \leq 1, \forall j \in J, \forall t \in T \tag{7}
\]

\[
x_{ijt} \in \{0,1\}, \forall i \in I, \forall j \in J, \forall t \in T \tag{8}
\]

Equation 1 presents the objective function of the model which is to minimize the average project risk \( R \). Constraint 2 limits the start time of the last activity to an upper limit. Constraint 3 limits the project cost to an upper limit. Constraint 4 limits the weighted average of the project quality to a lower bound. Constraint 5 preserves the precedence relationships of each activity in the project. Constraint 6 preserves the non-preemptive assumption of the model, where activities may only start once and cannot be interrupted. Constraint 7 preserves the resource requirements of each activity, where resources may not start another activity while an activity is ongoing. And finally, constraint 8 restricts the decision variables to a binary value.
D. Case Study

The small-scale problem presented by Pollack-Johnson and Liberatore (2006) shall be used to illustrate the MRCPSP-DTCQRT model.

![Case study for a construction project from Pollack-Johnson and Liberatore (2006)](image)

The authors presented a set of 12 activities with precedence relations illustrated in Figure 4. For this research, modes are assumed to be resources, and risk values are added to each resource and activity. Risk values were generated by subtracting quality values from the total of 100%, which highlights the inverse relationship between quality and risk, and randomly selecting a value between 10% to the result of the previous operation. Table 1 presents the resulting TCQR values for the case study.

Table 1. Available resources (modes) for each activity with time-cost-quality-risk (T, C, Q, R) values

<table>
<thead>
<tr>
<th>Activity</th>
<th>Resource 1</th>
<th>Resource 2</th>
<th>Resource 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3, 21600, 70, 30)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>(1, 7200, 70, 20)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>(4, 28800, 80, 20)</td>
<td>(3, 39600, 70, 20)</td>
<td>(4, 35000, 90, 10)</td>
</tr>
<tr>
<td>4</td>
<td>(6, 28800, 70, 20)</td>
<td>(3, 43200, 70, 10)</td>
<td>(4, 36000, 60, 20)</td>
</tr>
<tr>
<td>5</td>
<td>(3, 14400, 70, 20)</td>
<td>(2, 19200, 60, 30)</td>
<td>(3, 12000, 60, 20)</td>
</tr>
<tr>
<td>6</td>
<td>(4, 9600, 70, 30)</td>
<td>(2, 16800, 60, 20)</td>
<td>---</td>
</tr>
<tr>
<td>7</td>
<td>(5, 12000, 70, 20)</td>
<td>(4, 14400, 80, 20)</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>(6, 28800, 70, 10)</td>
<td>(3, 57600, 60, 20)</td>
<td>---</td>
</tr>
<tr>
<td>9</td>
<td>(3, 10800, 70, 20)</td>
<td>(2, 16800, 70, 30)</td>
<td>(3, 9000, 60, 30)</td>
</tr>
<tr>
<td>10</td>
<td>(3, 7200, 70, 20)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>11</td>
<td>(2, 4800, 60, 20)</td>
<td>(1, 6000, 70, 30)</td>
<td>(2, 1800, 50, 40)</td>
</tr>
<tr>
<td>12</td>
<td>(3, 14400, 70, 10)</td>
<td>(2, 19200, 70, 20)</td>
<td>(3, 16000, 80, 10)</td>
</tr>
</tbody>
</table>

To illustrate the model’s use with the case study, the Python-MIP package was implemented on Google Colaboratory. This open-source package uses the branch-and-cut method as the main solution methodology for mixed-integer linear programming models implemented on the software (Santos & Toffolo, 2020).

FINDING AND DISCUSSION

Table 2. Solutions to the small-scale case study using the MRCPSP-DTCQRT model

<table>
<thead>
<tr>
<th>Solution</th>
<th>Sorted by</th>
<th>Time (d)</th>
<th>Cost ($)</th>
<th>Quality (%)</th>
<th>Risk (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shortest duration</td>
<td>20</td>
<td>246400</td>
<td>68.33</td>
<td>20.00</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>21</td>
<td>241800</td>
<td>70.00</td>
<td>19.17</td>
</tr>
<tr>
<td>3</td>
<td>Lowest cost</td>
<td>35</td>
<td>181200</td>
<td>67.50</td>
<td>22.50</td>
</tr>
</tbody>
</table>
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Table 2 presents these non-dominated solutions for the case study. Plotting the results of the program on a resource-time graph provide a visual for project managers to validate the model's effectiveness in satisfying the constraints of the model.

Solutions 1 and 2 focus on solutions with the lowest time duration among the feasible schedules, as illustrated in Figure 5. Compared to others, the solutions with the shortest duration tend to have higher costs, lower quality, and high risk.

Solution 3 and 4 focus on solutions with the lowest cost as shown in Figure 6. Like the previous solutions, focusing on low costs negatively impact the other three goals with longer time durations, lower quality, and higher risks.

Solution 5 and 6 focus on solutions with the highest quality as illustrated in Figure 7. For this case study, higher quality results in lower costs compared to Solution 1 and 2 but result in longer time durations and higher risk.
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Figure 7. Schedules with highest quality

Solution 7 and 8 focus on solutions with the lowest risk seen in Figure 8. The project duration for these solutions is quite high compared to the other solutions.

Figure 8. Schedules with lowest risk

And finally, Solution 9 and 10 present compromise solutions that aim to balance time, cost, quality, and risk of the project as seen in Figure 9.

Figure 9. Schedules with compromised trade-off parameters

Plotting the solutions on a time-cost axis will yield the graph shown on Figure 10. Comparing to the hypothesized relationship of time-cost-quality-risk in Figure 3, there are trends with solutions of the highest quality and lowest risk taking the upper right side of the chart, and the reverse at the lower left.
A comparison of the solutions generated in this paper with the solutions generated by Pollack-Johnson and Liberatore (2006) would show that the MSRCSP-DTCQRTP is more realistic. First, the present model assigns modes to resources, meaning that a mode cannot perform two activities at the same time as evident in the previous study. Second, the inclusion of risk provides a more comprehensive set of solutions that seek to balance the four trade-off parameters not considered in Pollack-Johnson and Liberatore (2006). Risk as part of the model was considered by Amoozad Mahdiraji et al. (2021), but only considers normal and crash mode rather than human resources. In essence, the solutions generated from the MRCSP-DTCQRTP is expected to produce more practical schedules for project managers.

CONCLUSION AND FURTHER RESEARCH
This paper sought to establish the MRCSP-DTCQRTP as a model for generating schedules that consider risk in addition to the usual considerations of time, cost, and quality. First, to create the model, the trade-off relationship between time, cost, quality, and risk has been derived and illustrated in prior literature. A set of graphs with a time-cost axis were generated to show the relationship between the four trade-off parameters among each other. Second, the model formulation for the MRCSP-DTCQRTP was generated, which incorporates risk as part of the trade-off objectives and provides a resource-based perspective on the time, cost, quality, and risk values. And lastly, a demonstration of the model's use with the branch-and-cut method from the Python-MIP package was used with a case study from prior literature.

This paper provides a preliminary perspective on incorporating risk as part of the trade-off consideration. Hence, this leaves a lot of room for future research with this extension. First, there is an opportunity to scale the model for medium and large-sized problems as this study was limited to a dozen activities at most, given available computing resources. Second, the time-cost-quality-risk trade-off objectives may be fully represented as four objective functions rather than as constraints in the model. To solve this model, metaheuristic algorithms are required to come up with near-optimal solutions. Lastly, as variables begin to increase in size, metaheuristics may significantly slow down computation to arrive at solutions quickly. Hence, researchers have begun to use machine learning to hasten the performance of metaheuristic algorithms.

REFERENCES


