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Deterioration Item Inventory Policy with Sales Discount under Order Quantity Discount and Permissible Delay in Payments

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Abstract

Most of the items sold at supermarkets have an expiration date. While many suppliers offer discounts and allow non-cash payments during certain time intervals. Supermarkets has the opportunity to take this offer so that profits can be increased, but they are faced with the nature of goods that have a shelf life. The greater the number of items stored, the greater the risk of expiration. Therefore, it is necessary to regulate the right inventory control policy so that the total costs incurred can be minimized. In this paper, it will be developed a multi-item inventory model of deteriorating item with sales discounts, all unit quantity discount from supplier and permissible delay in payments. The aim is to determine the right policy for controlling the inventory of items that have an expiration date in order to minimize losses and total inventory costs. This model is expected to help supermarkets in determining the optimal number of orders and when to place orders so as to minimize total inventory costs. The model solution is carried out with an optimization approach based on the parameters that affect the model. A numerical example is given at the end of this paper for model validation and illustrates the model solving algorithm.

Keywords: inventory model, deterioration, sales discount, all unit quantity discount, permissible delay in payments

INTRODUCTION

Inventory is an idle resource, so its existence can be seen as a waste and this means a burden for a business unit in the form of higher costs. One product that often causes waste in inventory is a product that has an expiration date. Therefore, its existence needs to be minimized.

Pamella Supermarket is a retailer that sells various kinds of daily necessities. One of the most important parts in this supermarket is the food and beverage products section. As a manager, Pamella swalayan pays special attention and good supervision to this section because it concerns the product expiration period. Quality, health, and consumer safety are the basis of services that are very important to be considered by the manager in providing products to consumers. If this is ignored, it is very dangerous for consumers and results in losses for managers. Problems will arise when there are a number of products that have passed the expiration date and the product cannot be returned. Then the product must be destroyed and become Pamella's responsibility. Of course, if this happened it would be a loss for Pamella. Therefore, Pamella Supermarket has made a policy of providing price discounts on products that are approaching expiration in the hope that it will attract consumers so that it will increase purchasing power or consumer desire to buy, which in the end Pamella does not suffer a greater loss.

So far, Pamella Supermarket has collaborated with several suppliers. One form of cooperation is when the goods have been received by Pamela, the supplier allows payment within a certain period of time or there is a period of delay in payment to the supplier. In addition, the supplier also provides a discount if the purchase is made in a certain minimum quantity. So that the more products purchased in the amount specified by the supplier, the price per unit of goods will be lower. When the supplier offers a price discount for any of the reasons, the retailer need to choose whether it will buy or not based on the benefits achieved by utilizing the policy of allowing delays in payments by suppliers (Ouyang, 2009). So far, Pamella supermarkets often buy products in large quantities to benefit from discounts provided by suppliers. The problem is that the more products purchased, the more invested capital that cannot be used for other, more profitable purposes and the greater the risk of expired products. The fewer products

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available, the greater the possibility of inventory shortages. As a result, the greater the loss of opportunities for profit. Therefore, inventory control of goods that have an expiration date is very necessary to avoid losses that arise even greater due to the expiration of goods (Nafisa et al., 2016a).

There are three factors that need to be considered in controlling the inventory of goods at Pamela supermarkets, the first factor is that the product has an expiration date, the second factor is whether or not each order in large quantities is necessary to get the benefits obtained from discounts given by suppliers that are all unit discounts, and the purchase of products together (joint orders), the third factor is the allowable delay in payment of products by suppliers. Referring to these three factors, this research will develop a multi-item inventory model for products that are approaching expiration by considering discounts and payment delays. This research is an inventory model development based on a case study at Pamella Swalayan. It is hoped that this proposed model can help Pamella Supermarket in determining the optimal order quantity, when the order should be made, so as to minimize the total inventory cost.

LITERATURE REVIEW

Inventory has an important and strategic position, because it has a major impact on the company's performance. The investment value of inventories at the factory level ranges from 25 – 35% of the total assets owned. Meanwhile, at the wholesaler, distributor and retailer level, they are in the range of 15 - 90% of the total product cost (Tersine, 1994). Most of the goods managed by retailers are perishable goods and have a limited shelf life which accounts for more than 54% of the total store sales (Nafisa et al., 2016a).

The inventory problem of deteriorating items was first studied by Whitin (1957), then Ghare and Schrader (1963) concluded in their study that the consumption of the deteriorating items was closely relative to a negative exponential function of time. And then, research on inventory control for deterioration items was carried out intensively by several researchers, including Li et al. (2010), Nafisah et al (2016a, 2016b), Singh (2016), Khorasani & Almasifard (2017), Uthayakumar & Karuppasamy (2017), Pervin et al. (2018), and Babu et al (2020).

In the classic Economic Order Quantity (EOQ) model it is assumed that the buyer must pay for the goods as soon as the goods are received by the customer. However, it often happens that the supplier will give permission for a certain period of time to settle the amount owed to him for the goods supplied. Generally, there is no charge if the amount owed is settled within the allowable fixed settlement period. After the end of this period, interest will be charged. Goyal (1985) was first developed an EOQ model under the condition of permissible delay in payments. Then then followed by several other researchers such as Seifert (2016), Farhangi & Mehdizadeh (2016), Jaggi (2017), Behera (2017), Benkherouf & Gilding (2017), and Kumar & Uthayakumar (2019).

This paper will collab and modify several basic models of inventory control to make a multi-item inventory model of deteriorating item with sales discounts, all unit quantity discount from supplier and permissible delay in payments. The model in this study is the development of the model developed by Nafisah (2016a) and Nafisah et al (2016b) by considering the real system conditions that occur in Pamella supermarkets. The difference with the Nafisah model (2016a) is that in this study the discount considered is all unit quantity discount, while in Nafisah (2016a) it is an incremental discount.

This paper consists of five parts. The first contains an introduction, the second contains is literature review, the third part is mathematical formulation. Then the four and final part is a numerical example to illustrate the model and conclusions of the research.

MATHEMATICAL FORMULATION

A. Notations and assumptions

The parameter notation used in this model is as follows:

P_j : unit purchase cost applicable to orders whose lot size falls in the interval U_0 to U_{k+1}

D_i : annual demand rate of item i, $i = 1, 2, \ldots, n$
Q_{ikd} : quantity of product near the expiration date of item i
A : ordering cost for the joint order
h_i : annual holding cost per item per year excluding interest charges, $h_i = P_i I$
m_i : average inventory item i for a year
T : time interval between successive orders
Q_i : order quantity of item i
J_i : unit selling price of item i that is close to expiration
E_i : unit normal selling price per unit of item i
R_i : lost profit margin due to discount
t_1 : time interval before discount
t_2 : time interval after discount
U_0 : minimum quantity that can be purchase
U_{k+1} : maximum quantity
M_i : difference in profit margin lost after item i is discounted
t : permissible delay in settling accounts
I_d : interest which can be earned per $ in a year
I_c : interest charges per $ investment in stocks per year
B_1 : cost of interest charges per $ investment in stocks per year
B_2 : cost of interest which can be earned per $ in a year
PC : purchasing cost per year
OC : ordering cost per year
HC : holding cost per year
EC : expired treatment cost per year
TC : total annual variable cost

The assumptions and limitations used in this study are as follows:

1) Demand is deterministic
2) Lead time constant
3) Expiration date known
4) Lead time constant
5) Returns are not permitted by the supplier. Products that have expired will be destroyed
6) Time period is infinite
7) Products that are close to expiration are immediately sold out
8) The case is $T \geq t$

B. Mathematical model

The model that will be developed in this paper is a deterioration multi-item inventory model that considers sales discounts on products that are approaching expiration date, price discounts from suppliers in the form of all unit quantity discounts, and permissible delay in payment of products that have been received by Pamela supermarkets by the supplier. The proposed model is expected to produce an appropriate inventory control policy in determining the optimal product order quantity, when to place an order, and how long the delay in product payment has been received by Pamella so that the total inventory costs incurred can be minimized.

The objective function of this research is to minimize the total annual variable costs. Total annual variable costs (TC) consist of purchasing cost per year (PC), ordering cost per year (OC), holding cost per year (HC), expired treatment cost per year (EC), cost of interest which can be earned per year, and cost of interest charges in stocks per year.

1) Total purchasing cost per year

Suppliers offer price cuts for each item with an all-unit quantity discount policy. The unit purchase cost is defined as follows:

$$P_j = \begin{cases} P_0 \, for \, U_0 \le Q \le U_1 \\ P_1 \, for \, U_1 \le Q \le U_2 \\ \vdots \\ P_k \, for \, U_k \le Q \le U_{k+1} \end{cases}, \quad with \; P_0 > P_1 > \cdots > P_k \qquad (1)$$

The purchasing cost item i per year

$$PC_i = P_{ji} D_i$$

Total purchasing cost per year

$$PC = \sum_{i=1}^{n} P_{ji} D_i \qquad (2)$$

2) Total ordering cost per year

The product ordered is from the same supplier, so that a joint replenishment order is carried out. The ordering cost item i per year

$$OC_i = \frac{A}{T}$$

Total ordering cost per year

$$OC = \frac{nA}{T} \qquad (3)$$

3) Total holding cost per year

The holding cost item i per year is of the annual holding cost per unit (h_i) multiplied by the average inventory item i for a year (m_i).

$$HC_i = h_i . mh_i$$
$$= P_i . I$$

m_i = (average inventory item i per cycle) x (amount of cycle per year)

The average inventory item i per cycle is average inventory item i during t_1:

$$m_{t1} = \frac{(Q_i - Q_{ikd})}{2} . t_1$$

While, $t_1 = \frac{(Q_i - Q_{ikd})}{D_i}$

$$m_{t1} = \frac{\sqrt{(Q_i - Q_{ikd})^2 2D_i}}{(Q_i - Q_{ikd})^2 2Q_i}$$

$$m_i =$$

So that, the holding cost item i per year is

$$HC_i = \frac{P_i . I(Q_i - Q_{ikd})^2 2Q_i}{}$$

Total holding cost n item per year is

$$HC = \sum_{i=1}^{n} \frac{P_{ji}(Q_i - Q_{ikd})^2 2Q_i}{} \qquad (4)$$

4) Expired treatment cost per year

Expired treatment costs are costs that are lost due to discount treatment on products that are nearing expiration.

The normal sales are purchase price plus profit margin.

$$E_i = P_i + M_i$$

$$M_i = E_i - P_i$$

It is assumed that the lost profit margin due to discount treatment for products approaching expiration is x % of the profit margin at normal prices, then amount of profit margin lost due to discount is

$$R_i = xM_i \tag{5}$$

So that, unit selling price of item i after discount due to expiration is

$$J_i = E_i - R_i$$

The expired treatment cost item i per year is

$$EC_i = \left(\frac{Q_{ikd}}{2}.R\right)_i \frac{D_i}{Q_i}$$

Total expired treatment cost per year is

$$EC = \sum_{i=1}^{n} \frac{Q_{ikd}R_i}{2T} \tag{6}$$

5) Cost of interest charges in stocks per year

Case $T \geq t$

When items are sold, and before the replenishment account is settled, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed, and effectively the items still e of settling the replenishmen

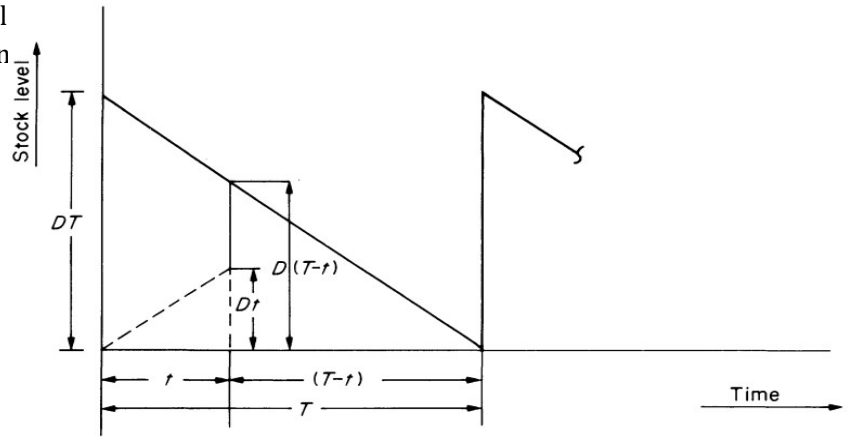

Fig. 1 Inventory position pattern with payment delay when $T \geq t$

The stock position subject to interest payable for item i during time $(T - t)$ per cycle is $D_i(T - t)$. The average stock during time $(T - t)$ is average stock per cycle which is equal to $\frac{D_i(T-t)}{2}.(T - t)$ or $\frac{D_i(T-t)^2}{2}$

Interest payable item i per cycle is $DPI\frac{(T-t)^2}{2}$

Interest payable item i per year is

$$B_{i1} = \frac{D_i\, P_i T I_c}{2} + \frac{D_i\, P_i t^2 I_c}{2T} - D_i\, P_i t I_c$$

Interest payable n item per year is

$$B_1 = \sum_{i=1}^{n} \left\{ \frac{D_i\, P_i T I_c}{2} + \frac{D_i\, P_i t^2 I_c}{2T} - D_i\, P_i t I_c \right\} \quad (7)$$

6) Interest earned during the permissible settlement period per year

Interest that can be earned per year due to allowable delays in payments. The maximum accumulated amount earning interest during the settlement period equals to

$$I = \begin{cases} Dtp, & \text{jika } T \geq t \\ DTp, & \text{jika } T < t \end{cases}$$

Case $T \geq t$

The stock position that earns interest for item i during time t per cycle is$D_i t$.

The average stock during time t is $\frac{D_i t^2}{2}$

Interest earned item i per cycle is $\frac{D_i\, P_i I_d\, t^2}{2}$

Interest earned item i per cycle per year is

$$B_{i2} = \frac{D_i\, P_i I_d\, t^2}{2T}$$

Interest earned n item per cycle per year are

$B_2 = \sum_{i=1}^{n} \left\{ \frac{D_i\, P_i I_d\, t^2}{2T} \right\}$ (8)

The total annual variable costs are

$$TC = PC + OC + HC + EC + B_1 - B_2$$

$$TC = \sum_{i=1}^{n} P_{ji}.D_i + \frac{nA}{T} + \sum_{i=1}^{n} \frac{P_i.I(Q_i - Q_{ikd})^2}{2Q_i} + \sum_{i=1}^{n} \frac{Q_{ikd}.R_i}{2T} + \sum_{i=1}^{n} \left\{ \frac{D_i\, P_i T I_c}{2} + \frac{D_i\, P_i t^2 I_c}{2T} - D_i\, P_i t I_c \right\} - \sum_{i=1}^{n} \left\{ \frac{D_i\, P_i I_d\, t^2}{2T} \right\} \quad (9)$$

Then $\frac{\partial TC}{\partial Q_{ikd}} = 0$ and $\frac{\partial TC}{\partial T} = 0$.

$$Q^*_{ikd} = \frac{Q_i.P_i.I - \frac{R_i \cdot D_i}{Q_i}}{P_i.I} \quad (10)$$

$$T^* = \sqrt{\frac{2nA + \sum_{i=1}^{n} \left(\frac{P_i.I.Q_{ikd}^2}{D} + Q_{ikd}.R_i + D_i\, P_{il} t^2 I_c - D_i\, P_i I_d\, t^2 \right)}{\sum_{i=1}^{n} \left(P_{ji}.D_i(I + I_c) \right)}} \quad (11)$$

While,

$Q_i^* = D_i.T$ (12)

C. Model Solution

The algorithm used in solving the developed model is as follows:

(1) Calculate the lost profit margin using equation (5)

(2) Calculate Q_i with all unit quantity discount

(a) Starting with the lowest unit cost, calculate $Q_i = \sqrt{\frac{2AD_i}{P_i.I}}$ at each unit cost until a valid EOQ is obtained.

(b) Calculate TC for the valid EOQ and all price-break quantities larger than the valid EOQ.

(c) Select the quantity with the lowest total cost in step (b) above.
(d) Do the same calculation for other items.

(3) Calculate Q^*_{ikd} for each price level based on the discount using equation (10), so that Q^*_{ikd} can be obtained based on the valid Q_i based on step (2). Do the same calculation for other items.
(4) Calculate T^* using equation (11) with Q^*_{ikd} based on step (3).
(5) Calculate Q_i^* using equation (12) with T^* step (4).
(6) Calculate TC using equation (9).

FINDING AND DISCUSSION

In this section, validation of the model will be carried out, namely by applying the model to real cases at Pamella Swalayan. The parameters used are as follows: considering three item of products, $D_1 = 3540$, $D_2 = 3750$, $D_3 = 4150$, $A = 7500$, $E_1 = 4325$, $E_2 = 5545$, $E_3 = 6375$, $x = 14\%$, $I_c = 0.12$, $_d = 0.07$, $I = 0.1$.
Prices offered on order sizes up to 120 and more than 120 units for each product are $P_{11} = 3500$, $P_{12} = 3125$, $P_{21} = 4350$, $P_{22} = 4000$, $P_{31} = 5200$, $P_{32} = 4750$, the case of $t > T$.

The calculation results obtained from the above case are the optimal order quantity, Q_i^* for each item is 437, 463 and 512 unit, the quantity of the discounted product because it is approaching the expiration date Q^*_{kdi} of each item is 54, 58, and 56 unit, while interval between orders, T^* is 45 days with total inventory cost per year is IDR 46,729,700.

Based on the calculation results, it means that the inventory control policy used so that the total cost of inventory incurred is minimum, Pamella must take advantage of discounts offered by suppliers, provide sales discounts to reduce the risk of expired products, and joint orders. In addition, the developed model is valid and in accordance with the conditions that occur in Pamella Swalayan.

CONCLUSION AND FURTHER RESEARCH

In this paper, an inventory model has been developed for a deterioration multi-item that considers sales discounts on products that are approaching expiration date, price discounts from suppliers, and permissible delay in payment. This research is an inventory model development based on a case study at Pamella Swalayan.

Based on the implementation of the model that has been carried out in the case being faced by Pamella, it is found that the model is valid. This means that this model can be used as a problem solving method in Pamella Swalayan. Through this model, Pamella can have an appropriate inventory control policy in determining optimal order quantity, sales discount quantity, and interval between orders so that the total inventory costs incurred can be minimized.

This model there are still many weaknesses. So for further research can consider several aspects, including that probabilistic demand, demand is a function of price, shortage, product return, etc.

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