



Metric-Based Reliability Transition Detection in Multisensor Localization

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Abstract

A two-dimensional range localization system can remain accurate over much of its operating range and then deteriorate rapidly as measurements degrade. This paper estimates the operating boundary instead of reporting only an average-error curve. Monte Carlo failure probabilities are fitted by a two-parameter logistic model whose midpoint defines the critical signal-to-noise ratio (SNR). Using the pooled fine-grid results from three independent random-seed sets, the weighted nonlinear least-squares experiment locates critical SNRs of 0.34, 2.93, and 6.05 dB for Gaussian, burst, and burst-plus-bias errors, respectively. Pairwise bootstrap tests confirm separation among the three transitions. Validation over three sensor layouts and five target positions preserves the degradation ordering while showing geometry-dependent critical-SNR ranges. A Huber estimator reduces the burst-related transition values without changing the Gaussian result materially. The contribution is an estimator-independent reliability diagnostic with explicit failure semantics, uncertainty intervals, and reproducible trial-level data; it is not a new localization optimizer.

Keywords: *Multisensor Localization, Reliability Transition, Failure Probability, Nonlinear Least Squares, Measurement Degradation.*

INTRODUCTION

Range-based localization is nonlinear, so moderate measurement degradation can cause a marked change in position error (Alanezi et al., 2021; Chan & Ho, 1994; Han et al., 2013; Luomala & Hakala, 2022). Sensor geometry, random errors, intermittent disturbances, and biased ranges may interact even when the estimator remains numerically stable (Abdulla, 2016; Beck et al., 2008; Gezici, 2008; Gustafsson & Gunnarsson, 2005; Patwari et al., 2003). Applications therefore need an operating boundary, not only an average accuracy value.

Root-mean-square error (RMSE) is useful for summarizing performance, but it does not directly report how often an application tolerance is violated. A reliability analysis should also measure the fraction of trials producing unacceptable estimates. A transition model can then associate that fraction with an interpretable critical SNR.

This paper makes four limited contributions. First, it converts an application-defined error tolerance into a fitted reliability transition with an uncertainty interval. Second, it separates Gaussian, burst, and burst-plus-positive-bias effects under controlled random seeds. Third, it checks the transition across three layouts, five target positions, and an alternative robust estimator (Chi et al., 2006). Fourth, it reports pointwise Wilson intervals (Robert & Casella, 2004) and bootstrap inference for transition locations. The contribution is the diagnostic workflow, not a new localization estimator or a universal SNR threshold.

The remainder of the paper reviews related localization and reliability analysis, defines the model and detection procedure, describes the simulation design, presents comparative results, and closes with demonstrated limitations and future validation needs.



LITERATURE REVIEW

Range-based localization studies commonly derive estimators, accuracy limits, or geometry-dependent error behavior. Nonlinear least squares and its damped variants provide transparent baselines (Björck, 1996; Marquardt, 1963; Nocedal & Wright, 2006). Robust losses reduce sensitivity to contaminated measurements. This distinction leads to the system model and transition definition presented next (Brown et al., 2001; Wilson, 1927).

Reliability reporting usually adds RMSE, percentiles, outage rates, or confidence intervals at a fixed operating condition. Those summaries do not by themselves estimate the SNR where an application-specific failure probability crosses a chosen level. Our narrower novelty is to fit that crossing, quantify its Monte Carlo uncertainty, refine it automatically, and test whether it persists across geometry, target position, error structure, and estimator choice. The detector can therefore sit above an existing localization algorithm rather than replace it.

Existing localization studies commonly characterize performance through estimation error, accuracy bounds, robustness to non-line-of-sight measurements, or convergence behavior. These analyses describe how accurately an estimator operates at selected noise levels. The present work addresses a different question: at which SNR does an application-defined failure event become more likely than success? The proposed diagnostic fits the complete failure-probability curve, reports its 0.5 crossing as a critical SNR, and quantifies the uncertainty of that crossing. Its novelty therefore lies in a reproducible reliability-transition analysis that can be applied to different estimators, rather than in a new localization estimator. This distinction leads to the system model and transition definition presented next.

RESEARCH METHOD

Range Measurement and Estimator

Five sensors at known coordinates measure the range to a stationary target. The measurement from sensor i is,

$$z_i = d_i(x) + e_i, \quad (1)$$

where z_i is the measured range, $d_i(x)$ is the Euclidean distance from the candidate target position x to sensor i , and e_i is the total measurement error. Depending on the case, this error contains Gaussian noise, a zero-mean burst, or a positive bias.

The position is estimated by weighted nonlinear least squares. Damped Gauss-Newton iterations use the analytic range Jacobian, a fixed centre initialization, bounded coordinates, and explicit finite-value and conditioning tests (Björck, 1996; Marquardt, 1963; Nocedal & Wright, 2006). Since all five sensors have the same nominal variance at a given SNR, the weights are equal within each trial (Hosmer et al., 2013; Huber & Ronchetti, 2009).

Error and Failure Metrics

For Monte Carlo trial r , localization error is,

$$E_r = \|\hat{x}_r - x_{true}\|_2. \quad (2)$$

This value is the Euclidean distance between the estimated and true target positions. A trial is marked as failed when E_r exceeds 10 m. Nonfinite estimates and solver nonconvergence are recorded as additional numerical failures. The 10 m failure threshold was specified in the simulation configuration before the Monte Carlo experiments and was not selected from the observed transition curves.

$$RMSE = \sqrt{[(1/R)\Sigma_r E_r^2]} , \quad P_f = \frac{N_{fail}}{R} . \quad (3)$$

RMSE gives more weight to large errors. Failure probability P_f is the proportion of trials that violate the fixed rule. Wilson 95% intervals describe binomial sampling uncertainty (Brown et al., 2001; Wilson, 1927).

Critical Transition Detection

The decreasing logistic failure curve is,

$$Pf(\gamma) = \frac{1}{[1 + \exp \exp(a(\gamma - \gamma_c))]} . \quad (4)$$

Here γ is SNR in decibels, γ_c is the critical SNR at which fitted failure probability equals 0.5, and $a > 0$ is the decrease in failure probability per decibel. A larger a indicates a sharper transition. The corresponding 10%-to-90% width is $2 \ln(9)/a$. Parameters are estimated by variance-scaled nonlinear least squares on binomial proportions. The largest probability drop on the coarse grid selects a case-specific one-decibel fine grid. Agreement is accepted when γ_c lies in the empirical 0.4-to-0.6 failure region. A parametric binomial bootstrap provides a 95% interval for γ_c (Hosmer et al., 2013; Huber & Ronchetti, 2009).

Reproducible Detection Procedure

The procedure has five stages. A fixed error tolerance first converts every localization estimate into a success or failure. A broad SNR scan then identifies the largest empirical probability drop. A one-decibel grid expands that interval by 2 dB on both sides. The logistic model is fitted to the fine-grid proportions and bootstrapped. Independent seed sets and sensitivity scenarios finally test whether the reported ordering survives sampling and configuration changes. The following experiments use this procedure to test whether the transition remains identifiable under changes in error structure, geometry, target position, and estimator.

Experimental Setup

The 100 m by 100 m area contains sensors at (5, 5), (95, 5), (95, 95), (5, 95), and (50, 12) m. The main target is at (52, 58) m. The fixed initial estimate is (50, 50) m. Figure 1 shows the geometry.

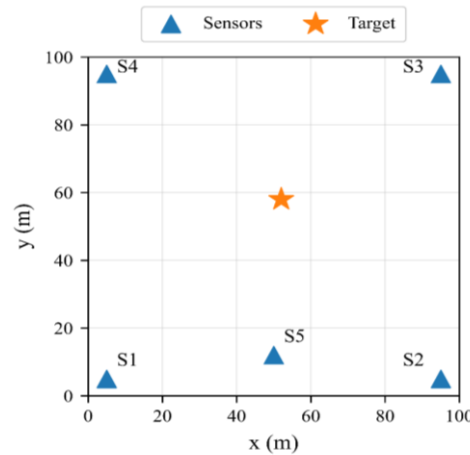


Figure 1. Five-sensor geometry and main target position.

Let

$$\varepsilon_i \sim N(0, \sigma^2) \quad B_i \sim (0.12) \quad U_i \sim N(0, (4\sigma)^2) \quad \} \quad (5)$$

and,

$$C_i \sim (0.18) \quad , \quad (6)$$

independently across sensors and trials. Gaussian error is,

$$e_i = \varepsilon_i \quad , \quad (7)$$

burst error is,

$$e_i = \varepsilon_i + B_i U_i \quad , \quad (8)$$

burst-plus-bias error is,

$$e_i = \varepsilon_i + B_i U_i + 3\sigma C_i \quad . \quad (9)$$

here,

$$\sigma = 14 \times 10^{-\gamma/20} \text{ m}. \quad (10)$$

The probabilities and multipliers create sparse, moderate contamination for a controlled stress test; they are not fitted to field data. Scaling every component with σ avoids an artificial high-SNR error floor. The 10 m tolerance corresponds to 10% of the simulated area's side length and approximately 7.1% of its diagonal. It serves as an application-level criterion for this controlled study rather than a universal localization standard. Results for 5 m and 15 m thresholds are also evaluated to quantify the sensitivity of the inferred transition to this design choice. The coarse SNR interval from -5 to 25 dB was selected to include both failure-dominated and predominantly reliable regimes under all tested error models. Each validation curve brackets a failure probability of 0.5 within this interval. A case-specific one-decibel fine grid was then used around the largest empirical probability decrease to improve the resolution of the transition estimate.

Every main experiment SNR uses 600 trials. Each largest-slope interval is expanded by 2 dB and sampled at 1 dB. Three base seeds are used: 20260804, 20261804, and 20262804. Trial seeds are deterministic functions of base seed, case, SNR, and trial index. The solver uses at most 75 iterations, a 10^{-6} m step tolerance, damping 10^{-3} , and a 25 m maximum step.

For additional validations, generalizability is tested with the baseline layout, a perimeter-cross layout [(50,5), (95,50), (50,95), (5,50), (20,20)], and a deliberately weak lower-edge layout [5,5), (27.5,5), (50,5), (72.5,5), (95,5)]. Each layout uses the main target and four additional targets at (25,25), (75,25), (75,75), and (25,75) m. Every coarse-grid point uses 400 trials for each of three seeds. Estimator sensitivity compares the original WNLS method with bounded Huber-loss range fitting using the standard 1.345 scale constant.

FINDINGS AND DISCUSSION

Coarse Error Behavior

RMSE decreases across the coarse SNR grid in every degradation case, as shown in Figure 2. High-SNR performance approaches a stable low-error region. At any fixed SNR, the structured disturbance cases generally produce larger RMSE than Gaussian noise alone. The 10 m horizontal line gives the physical scale of the trial failure threshold; it is not used as an RMSE decision rule.

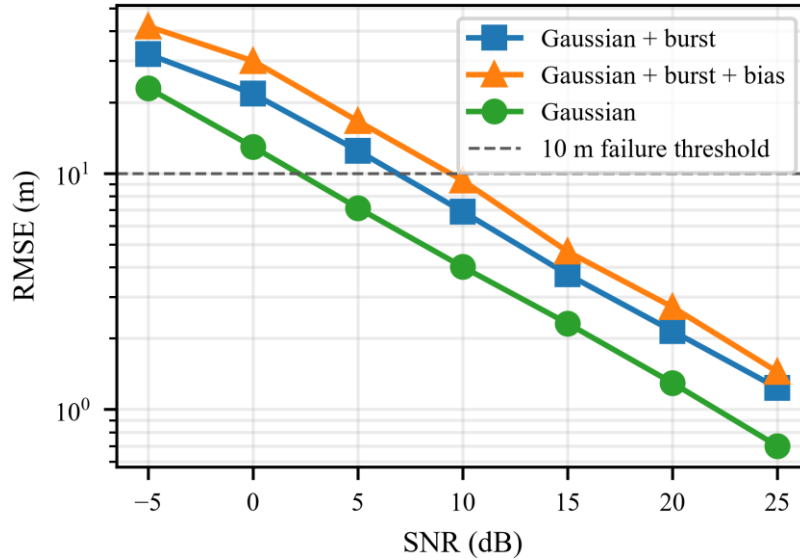


Figure 2. Coarse-scan RMSE for the three measurement cases.

Table 1. Selected coarse-scan localization results.

Case	SNR	RMSE	Median	P90	P_f
Gaussian	0	12.94	10.19	19.32	0.515
Gaussian	5	7.13	5.93	10.94	0.142
Gaussian	10	4.01	3.33	5.98	0.003
Gaussian	15	2.31	1.90	3.48	0.000
Gaussian + burst	0	21.72	14.05	37.49	0.698
Gaussian + burst	5	12.46	7.85	18.63	0.362
Gaussian + burst	10	6.88	4.22	11.16	0.120
Gaussian + burst	15	3.73	2.30	5.91	0.015
Gaussian + burst + bias	0	29.91	21.78	50.10	0.840
Gaussian + burst + bias	5	16.63	11.35	24.19	0.555
Gaussian + burst + bias	10	9.32	6.28	14.72	0.227
Gaussian + burst + bias	15	4.66	3.58	7.00	0.030

Failure Transition

All primary transition values reported in the abstract, main-results table, estimator comparison, and discussion are obtained from the same pooled fine-grid dataset containing 1,800 trials per SNR across three seed sets. Coarse-grid validation results are used only for the additional geometry and target scenarios. For the baseline layout and main target, the pooled primary estimate is reused in the generalizability summary to avoid presenting two estimates for the same configuration. The fitted critical SNR is 0.37 dB for Gaussian noise, 3.09 dB for Gaussian noise with bursts, and 6.06 dB for bursts plus positive bias. The ordering is physically interpretable: more structured measurement corruption requires a cleaner nominal operating point before fewer than half of the trials fail.

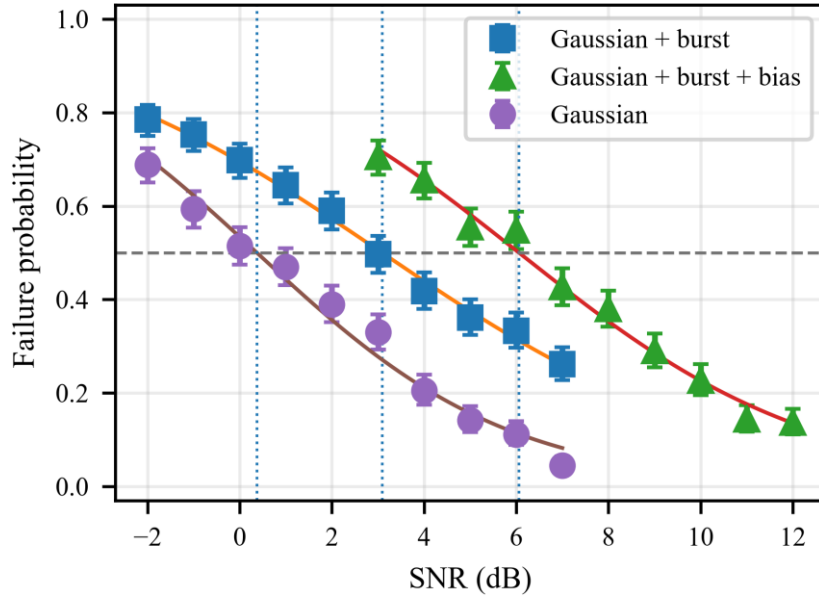


Figure 3. Empirical fine-scan failure probabilities, Wilson intervals, and logistic transition curves.

Table 2. Logistic transition estimates for the accepted base seed.

Case	γ_c (dB)	Width (dB)	95% CI (dB)	R^2
Gaussian	0.37	12.09	0.18-0.55	0.982
Gaussian + burst	3.09	16.47	2.89-3.29	0.995
Gaussian + burst + bias	6.06	14.09	5.86-6.24	0.988

All three fitted midpoints lie inside their empirical 0.4-to-0.6 transition regions. The coefficients of determination range from 0.982 to 0.995. Transition widths from 12.09 to 16.47 dB show that the change is progressive rather than an abrupt physical phase transition.

Seed Stability and Numerical Checks

Across the three seed sets, critical SNR ranges are 0.22-0.41 dB for Gaussian noise, 2.78-3.09 dB for bursts, and 6.01-6.08 dB for bursts plus bias. Corresponding standard deviations are 0.10, 0.16, and 0.04 dB. This variation is small relative to the separation among degradation cases.

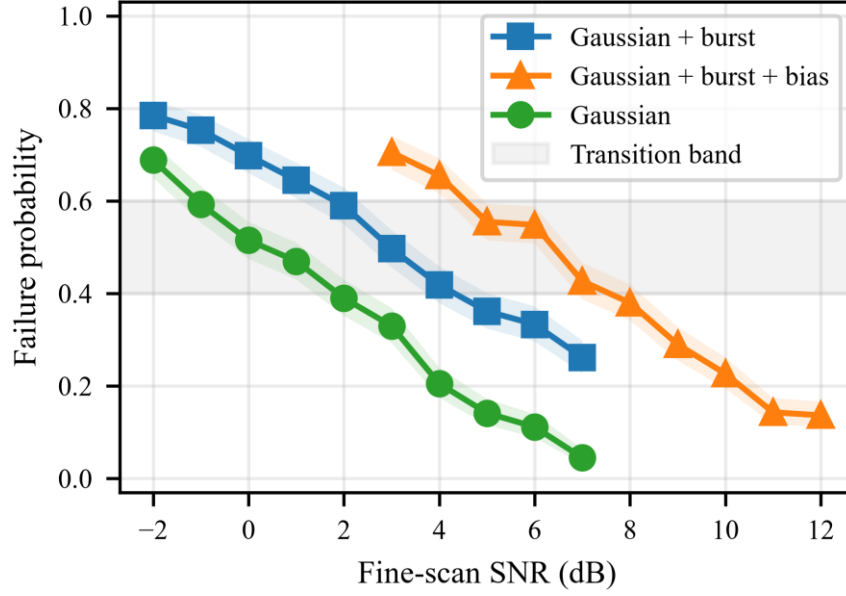


Figure 4. Detailed one-decibel transition-region probabilities with Wilson intervals.

Numerical nonconvergence increases at the most degraded coarse SNR, reaching approximately 10% in the burst cases at -5 dB. It never dominates the application failures and is small around the fitted transitions. Re-running the accepted configuration reproduces all trial seeds, tables, and figures.

Statistical Separation of Transitions

Pooling the three seed sets gives 1,800 trials per fine-grid SNR. A 1,000-replicate parametric bootstrap estimates the burst-minus-Gaussian transition difference as 2.60 dB (95% CI 2.44-2.76), burst-plus-bias minus burst as 3.11 dB (2.95-3.28), and burst-plus-bias minus Gaussian as 5.71 dB (5.57-5.86). All intervals exclude zero; the resolution-limited two-sided bootstrap p-value is 0.002 for each comparison. The inference concerns Monte Carlo sampling under the specified model.

Geometry and Target Generalizability

Across 15 geometry-target scenarios, the fitted critical SNR ranges from -0.28 to 2.33 dB for Gaussian error, 2.33 to 5.00 dB for burst error, and 5.61 to 7.94 dB for burst plus bias. Every fit bracket $Pf = 0.5$ within the tested SNR range. The degradation ordering persists, but the 2.3-2.7 dB within-case spans confirm that the numerical boundary is geometry- and target-dependent.

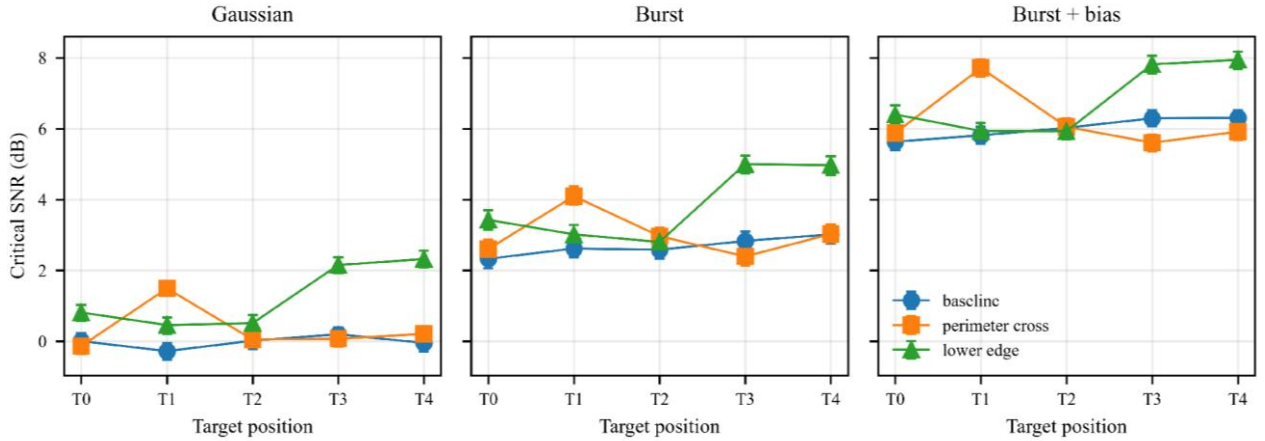


Figure 5. Critical-SNR sensitivity across three sensor layouts and five target positions; bars show bootstrap 95% intervals.

Table 3. Critical-SNR ranges across target positions for each layout (dB).

Layout	Gaussian	Burst	Burst + bias
baseline*	-0.27 to 0.20	2.33 to 3.02	5.64 to 6.31
perimeter cross	-0.13 to 1.50	2.40 to 4.10	5.61 to 7.72
lower edge	0.46 to 2.32	2.80 to 5.00	5.93 to 7.94

*Baseline-T0 denotes the primary geometry and target and therefore reuses the pooled fine-grid estimate. The remaining geometry-target combinations are estimates from the validation scans.

Alternative Estimator Comparison

The Huber estimator leaves the Gaussian transition nearly unchanged but moves the burst and burst-plus-bias boundaries toward lower SNR. This is consistent with down-weighting isolated large residuals. The transition detector therefore remains meaningful when the localization algorithm changes and can quantify the reliability benefit of robust estimation.

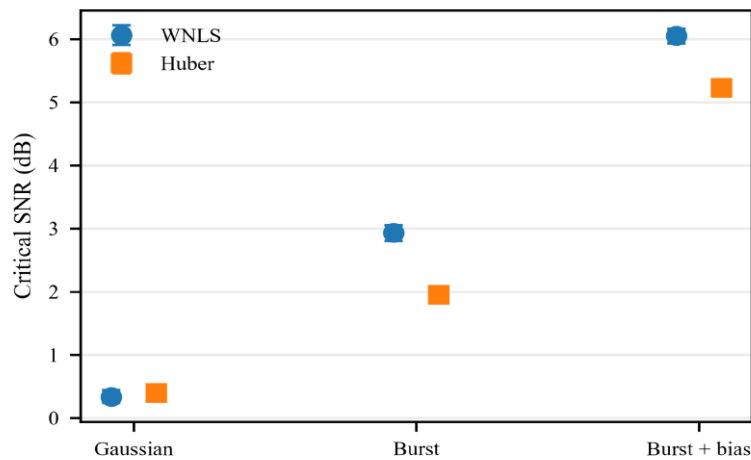


Figure 6. Critical SNR for WNLS and Huber estimators using all three seed sets; bars show bootstrap 95% intervals.

Table 4. Aggregated estimator comparison across three seed sets.

Estimator	Case	γ_c (dB)	95% CI (dB)
WNLS	Gaussian	0.34	0.24-0.44
WNLS	Gaussian + burst	2.93	2.81-3.05
WNLS	Gaussian + burst + bias	6.05	5.93-6.16
HUBER	Gaussian	0.40	0.30-0.51
HUBER	Gaussian + burst	1.95	1.85-2.05
HUBER	Gaussian + burst + bias	5.22	5.09-5.35

Failure-Threshold Sensitivity

Changing the application tolerance shifts the transition as expected but does not change the ordering of error cases. This check supports 10 m as a transparent design choice rather than an outcome-tuned constant; it does not make that value universal.

Table 5. Critical SNR under alternative failure thresholds (dB).

Threshold	Gaussian	Burst	Burst + bias
5 m	6.38	8.71	11.80
10 m	0.26	2.90	5.98
15 m	-2.80	-0.66	2.72

CONCLUSIONS

This study introduced a metric-based procedure for identifying the SNR at which a multisensor localization system transitions from predominantly successful operation to failure-dominated behavior. The procedure converts an application-defined localization tolerance into an empirical failure probability and fits this behavior using a logistic model. The fitted midpoint provides an interpretable critical SNR, while the bootstrap confidence interval quantifies Monte Carlo sampling uncertainty. These diagnostic complements conventional RMSE analysis by describing the operating boundary of the system instead of reporting only average localization accuracy at individual SNR values. Using the pooled fine-grid results from three random-seed sets, the WNLS estimator produced critical SNR values of 0.34, 2.93, and 6.05 dB under Gaussian, burst, and burst-plus-bias errors, respectively. The confidence intervals and pairwise bootstrap comparisons support a clear separation between these transitions under the specified simulation model. Sparse burst contamination shifts the reliability boundary toward higher SNR, while the addition of a positive bias produces a further degradation. The same ordering was retained across the tested sensor layouts and target positions. The numerical transition values changed with geometry, confirming that the metric should be interpreted as a configuration-dependent operating characteristic rather than a universal constant. The estimator comparison further showed that the proposed diagnostic is not restricted to WNLS localization. Huber fitting left the Gaussian transition nearly unchanged but moved the burst and burst-plus-bias transitions toward lower SNR values. This result illustrates how the critical-SNR metric can quantify the reliability benefit of an

alternative estimator under contaminated measurements. The threshold-sensitivity analysis produced different numerical boundaries for 5 m, 10 m, and 15 m tolerances while preserving the degradation ordering. The transition estimate therefore depends on the selected application tolerance, as expected, yet still provides a consistent basis for controlled comparisons.

LIMITATION AND FURTHER RESEARCH

The study remains simulation-only. Sensor errors are independent, burst and bias magnitudes scale with Gaussian standard deviation, and geometry is perfectly known. Field multipath, correlated obstruction, calibration error, timing drift, motion, and high-SNR bias floors are not modelled. The added layouts and targets demonstrate sensitivity rather than deployment validity. Equal sensor variances also make WNLS equivalent to ordinary nonlinear least squares within a trial. The bootstrap intervals and pairwise tests quantify Monte Carlo sampling uncertainty under the specified simulation model. They do not account for model misspecification, correlated sensor disturbances, calibration uncertainty, or variation across physical environments.

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